Solutions to Problems 1

1. From

$$\hat{j}_-|j,m+1\rangle = \sqrt{(j-m)(j+m+1)}|j,m\rangle \ , \ \hat{j}_+|j,m\rangle = \sqrt{(j-m)(j+m+1)}|j,m+1\rangle$$

construct the matrices of the operator $\hat{j}$ for $j=\frac{1}{2}$ (the Pauli matrices) and $j=1$.

$j=\frac{1}{2}$

$$j_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ \text{-}1 & 0 \end{pmatrix} \quad j_y = \frac{1}{2}\begin{pmatrix} 0 & \text{-}i \\ i & 0 \end{pmatrix} \quad j_z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{1}$$

$j=1$

$$j_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad j_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \text{-}i & 0 \\ i & 0 & \text{-}i \\ 0 & i & 0 \end{pmatrix} \quad j_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

2. Use the explicit form of $Y_{20}(\theta,\phi)$ and $Y_{40}(\theta,\phi)$.

3. $s=1,3$.

4. ^{1}SDGI; ^{3}PFH.

5. $p(m_s=\frac{1}{2}) = (21\frac{1}{2}\frac{1}{2}|\frac{5}{2}\frac{3}{2})^2$; $p(m_s=-\frac{1}{2}) = (22\frac{1}{2}-\frac{1}{2}|\frac{5}{2}\frac{3}{2})^2$.

6. $(\lambda=2)^2$: $J^\pi = 0^+,2^+,4^+$; $(\lambda=2)^3$: $J^\pi = 0^+,2^+,3^+,4^+,6^+$; $(\lambda=3)^2$: $J^\pi = 0^+,2^+,4^+,6^+$; $(\lambda=3)^3$.

7. (a)

$$|j_1(j_2j_3)J_{23};JM\rangle = \sum_{J_{12}}(-1)^{j_1+j_2+j_3+J}\sqrt{(2J_{12}+1)(2J_{23}+1)}\begin{Bmatrix} j_1 & j_2 & J_{12} \\ j_3 & J & J_{23} \end{Bmatrix}|(j_1j_2)J_{12}j_3;JM\rangle \tag{3}$$

(b)

$$|(j_2j_3)J_{23},j_1;JM\rangle = (-1)^{j_1+J_{23}-J}|j_1,(j_2j_3)J_{23};JM\rangle \tag{4}$$

(c)

$$|j_3,(j_2j_1)J_{12};JM\rangle = (-1)^{j_1+j_2+j_3-J}|(j_1j_2)J_{12},j_3;JM\rangle \tag{5}$$

8.

$$|s_{1/2}d_{3/2};J=2\rangle \equiv |0\tfrac{1}{2}(\tfrac{1}{2}),2\tfrac{1}{2}(\tfrac{3}{2});J=2,M\rangle = \sum_{S=0,1}\sqrt{40(2S+1)}\begin{Bmatrix} 0 & 2 & 2 \\ \frac{1}{2} & \frac{1}{2} & S \\ \frac{1}{2} & \frac{3}{2} & 2 \end{Bmatrix}|02(2)\tfrac{1}{2}\tfrac{1}{2}(S);J=2,M\rangle \tag{6}$$