Lecture 6. Solutions to Problems.

1. Use formulae of Lecture 5 for the calculation of the matrix elements of the irreducible tensor operators.

2.
$$\langle r^L \rangle = \int R^*_{n_f l_f}(r) r^{L+2} R_{n_i l_i}(r) dr = \frac{3R_0^{2L}}{L+3} \ , \tag{1}$$

$$B(\mathcal{E}L; j_i = L + 1/2 \to j_f = 1/2) = e^2 \frac{(1.2)^{2L}}{4\pi} \left(\frac{3}{L+3}\right)^2 A^{2L/3} \ \mathrm{fm}^{2L} \ . \tag{2}$$

3.
$$B(\mathcal{E}2; 1/2^+ \to 5/2^+) \approx 21.56 \ e^2_{eff} \ \mathrm{fm}^4 \ . \tag{3}$$
The effective charges are $e_{eff}(\nu) = 0.54 \ e$ and $e_{eff}(\pi) = 1.72 \ e$

4.
$$Q(^9\mathrm{Li}(3/2^-)) = -\frac{2}{5} e \langle p_{3/2} | r^2 | p_{3/2} \rangle \approx -2.5 \ \mathrm{e.fm}^2 \ . \tag{4}$$

5.
$$\mu(j) = \begin{cases} j g_l + (g_s - g_l)/2 \ (\mu_N) & \text{for} \quad j = l + 1/2 \\ j g_l - j(j+1)(g_s - g_l)/2 \ (\mu_N) & \text{for} \quad j = l - 1/2 \end{cases} \tag{5}$$

6.
$$\begin{aligned} &\mu(^{31}\mathrm{Al}(5/2^+)) \approx 4.8 \ (\mu_N) \ , \\ &Q(^{31}\mathrm{Al}(5/2^+)) = -\tfrac{4}{7} e \langle d_{5/2} | r^2 | d_{5/2} \rangle \approx -7.0 \ \mathrm{e.fm}^2 \ . \end{aligned} \tag{6}$$

7. $\varepsilon(\nu p_{3/2}) = 1.943$ MeV, $\varepsilon(\nu f_{5/2}) = 2.575$ MeV, $\varepsilon(\nu p_{1/2}) = 3.614$ MeV, $\varepsilon(\pi p_{3/2}) = 1.716$ MeV, $\varepsilon(\pi f_{5/2}) = 2.588$ MeV, $\varepsilon(\pi p_{1/2}) = 3.475$ MeV.

8. $\varepsilon(\nu s_{1/2}) = 1.205$ MeV, $\varepsilon(\nu g_{7/2}) = 1.882$ MeV, $\varepsilon(\nu d_{3/2}) = 2.042$ MeV, $\varepsilon(\nu h_{11/2}) = 2.170$ MeV.

9. The single-particle energies for neutrons and protons: $\varepsilon(\nu i_{11/2}) = 0.779$ MeV, $\varepsilon(\nu j_{15/2}) = 1.423$ MeV, $\varepsilon(\nu d_{5/2}) = 1.567$ MeV, $\varepsilon(\nu s_{1/2}) = 2.032$ MeV, $\varepsilon(\nu g_{7/2}) = 2.491$ MeV, $\varepsilon(\nu d_{3/2}) = 2.538$ MeV, $\varepsilon(\pi f_{7/2}) = 0.896$ MeV, $\varepsilon(\pi i_{13/2}) = 1.609$ MeV, $\varepsilon(\pi f_{5/2}) = 2.826$ MeV, $\varepsilon(\pi p_{3/2}) = 3.120$ MeV.
The single-hole energies for neutrons and protons: $\varepsilon(\nu f^1_{5/2}) = 0.570$ MeV, $\varepsilon(\nu p^{-1}_{3/2}) = 0.898$ MeV, $\varepsilon(\nu i^{-1}_{13/2}) = 1.633$ MeV, $\varepsilon(\nu f^{-1}_{7/2}) = 2.340$ MeV, $\varepsilon(\nu h^{-1}_{9/2}) = 3.414$ MeV, $\varepsilon(\pi d^{-1}_{3/2}) = 0.351$ MeV, $\varepsilon(\pi h^{-1}_{11/2}) = 1.348$ MeV, $\varepsilon(\pi d^{-1}_{5/2}) = 1.683$ MeV, $\varepsilon(\pi g^{-1}_{7/2}) = 3.474$ MeV.